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Optimization Models of Tool Path Problem for CNC Sheet Metal Cutting Machines

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Abstract: The problem of tool path optimization for CNC sheet metal cutting equipment is considered. Sheet metal cutting equipment includes laser/plasma/gas/water-jet machines and some others. Users of CAD/CAM systems develop numerical control programs for the cutting equipment after nesting of parts onto the sheet. The control programs contain information about tool path. The tool path is a routing of cutter head used for cutting of sheet material. Classification and the correspondent mathematical models of tool path problem are considered. The tasks of cost/time minimization for various types of cutting techniques are formalized. Mathematical formalization of technological constraints for these tasks is also described. Unlike the known analogs this formalization allows to consider constraints of thermal cutting. In some cases the optimization tasks can be interpreted as discrete optimization problem (generalized travel salesman problem with additional constraints, GTSP). In paper also the developed exact algorithm and some heuristic algorithms of tool path optimization based on described models is reported. Results of computing experiments for some instances are given

Keywords: Tool path problem, CNC sheet metal cutting machines, control programs, technological constraints, thermal cutting, discrete optimization, GTSP

1. INTRODUCTION

In various industries many parts are produced from sheet materials by CNC equipment. Such kind of equipment includes, for instance, machines for laser, plasma, gas, and water-jet cutting. Special software (Computer-Aided Manufacturing, CAM systems) provides an automation of development of NC (numerical control) programs. Generating of NC programs is next step after nesting that is positioning parts onto sheet material. Optimization of sheet utilization reduces the cost of sheet material used for parts producing. The nesting problem was not considered in this study. The control programs contain information about tool path for CNC machine and some technological commands. Optimization of tool path reduces time and cost of cutting process. First classification of problem was conducted by Hoeft and Palekar (1997). Tool path problems are usually divided into 4 classes depending on cutting technique and its parameters (see, for Example, Dewil et al. (2015)):

1. *Continuous Cutting Problem (CCP).*
2. *Endpoint Cutting Problem (ECP).*
3. *Intermittent Cutting Problem (ICP).*
4. *Generalized Traveling Salesman Problem (GTSP).*

Petunin (2015) offered new classification of cutting techniques and described one more class of problem: *Segment Continuous Cutting Problem (SCCP).*

The tool path includes the following components (see Fig.1, Fig.2):

- pierce points (piercings);
- points of switching the tool off;
- tool trajectory from piercing upto point of switching the tool off;
- lead-in (tool trajectory from piercing upto the entry point on the equidistant contours);
- lead-out (tool trajectory from exit point on equidistant contour upto tool switching off point);
- Airtime motions (linear movement from tool switching off point upto the next piercing).

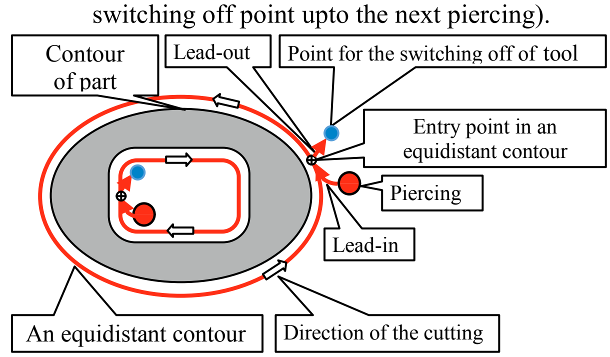

Figure 1. Scheme of the standard cutting technique

Fig. 2 shows an example of non-standard cutting techniques. In practice CAM systems users often use the various cutting technique interactively to get technologically admissible solutions. Trajectories of lead-in to the contour and lead-out (exit of contour) also can be different (along the straight line, along the arc, “in corner”

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and etc.). Decrease of the sheet material deformation is provided in particular by lead-in "in corner" of part (Fig.3).

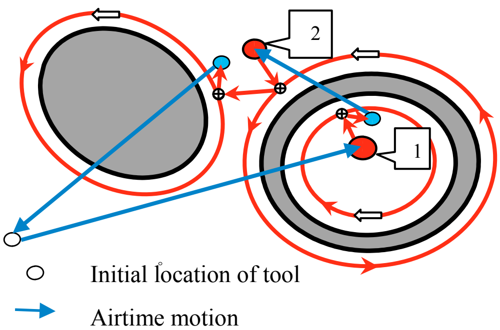

Figure 2. Example of cutting for two parts (three contours) by using technique "chain cutting"

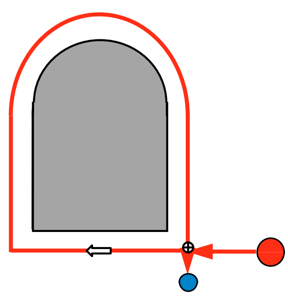

Figure 3. Example of lead-in "in corner"

Automatic methods are developed generally to discrete models (GTSP and ECP). Some heuristic algorithms are offered by Lee and Kwon (2006), Verhoturov and Tarasenko (2008), Xie and al. (2009), Yang et al. (2010), Dewil et al. (2011, 2015, 2015a), Jing and Zhige (2013), Helsgaun (2014). For GTSP class without technological constraints the effective approximate algorithms offered by Karapetyan and Gutin (2011, 2012) also can be used. For the same class of problems with precedence constraints Petunin and al. (2014, 2015a) described an exact algorithm based on method of dynamic programming.

The existing mathematical models and algorithms do not consider many technological constraints of the thermal cutting process in particular heuristic rules “part hardness rule” and “sheet metal hardness rule”. The latter was described by Petunin (2009). In this paper we formalize this kind of constraints and describe a new formalization of the tool path problem concerning cutting technique we used. In certain cases we will interpret the considered optimizing tasks as problems of discrete optimization with additional constraints. In paper the results of computing experiments for some instances are also given

2. CLASSIFICATION OF THE CUTTING TECHNIQUES AND FORMAL DEFINITION OF TOOL PATH

Definition 1. Segment of cutting $S = MM^*B^S$ is a tool trajectory from piercing M upto point of switching the tool off M^*. $(S \subset \mathbb{R}^2; M = (x, y), M^* = (x^*, y^*) \in \mathbb{R}^2)$.

Definition 2. Basic segment B^S is a part of segment $S = MM^*$ without trajectory lead-in and trajectory lead-out.

Let's consider that unlike a cutting segment the corresponding basic segment has no direction of cutting, i.e. it contains only geometry information. In Fig. 4 (see also Fig.2) two basic segments are allocated with dashed lines of orange and yellow color.

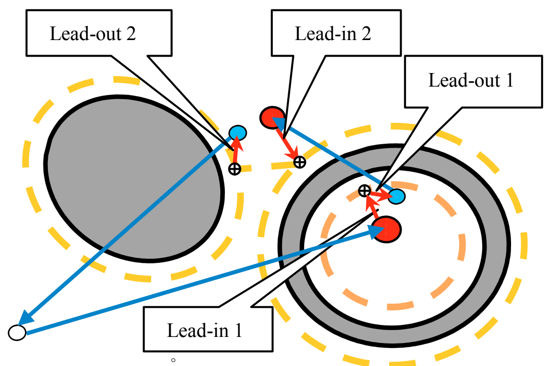

Figure 4. Illustration of term “basic segment” for example given in Fig.2

All the cutting techniques we divide into three classes:

1. Standard cutting.
2. Multi-contour cutting.
3. Multi-segment cutting.

Standard cutting technique assumes:

- Piercings number is equal to contours number and parts number;
- Cutter head runs each closed equidistant contour of part to cut exactly once from beginning to end.

At the same time the basic segment coincides with this closed contour.

The multi-contour cutting cuts several contours in one segment of cutting. External contours of parts are cut jointly with the only piercing without switching cutter head off.

The multi-contour cutting can be itself divided into 2 classes: “chain” cutting (see Fig.2), and multi-section cutting. The latter assumes that some contours can be cut piecemeal. Example of Multi-section cutting is in Fig.5.

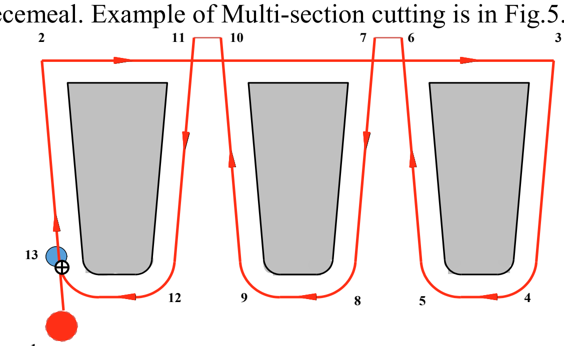

Figure 5. Multi-section cutting: “Snake” technique

Multi-segment cutting cuts single contour in several segments of cutting (Fig. 6).

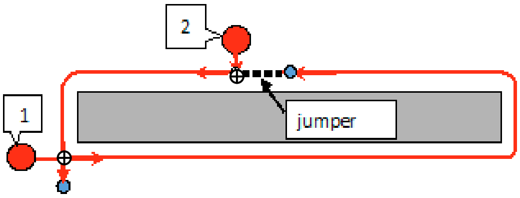

Figure 6. Multi-segment cutting: “Jumper” technique

Let $A_1, A_2, \ldots, A_n$ to be finite set of two-dimensional geometrical objects. These objects are geometrical models of flat parts. Each object is described by one or several closed curves (boundary contours). Let also N to be number of external and the internal closed contours $C_1, C_2, \ldots, C_N$, that describe parts positions (the nesting) on sheet material. $(A_i, C_j \subset \mathbb{R}^2; i = \overline{1,n}; j = \overline{1,N}; N \geq n)$. Example of nesting is shown in Fig. 7 (n=18, N=23).

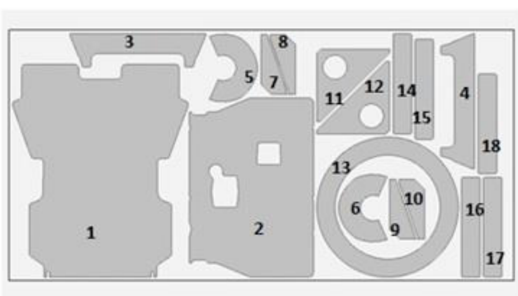

Figure 7. Nesting example with parts containing internal contours

Let K is a number of segments the tool path consists of. $S_k = M_k M_k^*; k = \overline{1,K}$. Single segment may contain one contour, a few contours (for the multi-contour cutting), or a part of contour (for multi-segment cutting). Sequence of segments is a permutation $i_1, i_2, \ldots, i_K$, i.e. the ordered set of natural numbers from 1 to K or bijection on a set $\{\overline{1,K}\}$. Thus, the tool path is defined by a tuple:

$$ROUTE = \langle M_1, M_1^*, \ldots, M_K, M_K^*, i_1, \ldots, i_K \rangle \qquad (1)$$

Fig.8 shows the scheme of tool path route for the nesting given on fig. 7. The tool path contains 21 segments. For cutting of external contours of parts 7 and 8, and also of 9 and 10 the Multi-contour cutting is used (brown color). All other 19 contours are cutting by the standard cutting.

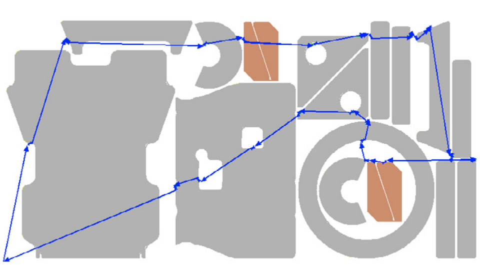

Figure 8. Example of tool path consisting of 21 cutting segments

During development of NC programs for CNC sheet metal cutting machines the problems of tool path optimization arise. As optimization criteria in these problems the parameters of cutting time t_{cut} and cost $F_{\cos t}$ are considered. They are calculated by following formulas:

$$t_{cut} = L_{on} \div V_{on} + L_{off} \div V_{off} + N_{pt} \cdot t_{pt}, \qquad (2)$$

$$F_{\cos t} = L_{off} \cdot C_{off} + L_{on} \cdot C_{on} + N_{pt} * C_{pt} \qquad (3)$$

Here L_{off} is length of idling tool path; L_{on} is length of working tool path; V_{off} is speed of idling tool path; V_{on} is speed of the working tool path; C_{off} is cost of idling tool path unit; C_{on} is cost of working tool path unit; N_{pt} is numbers of piercing; t_{pt} is time of one piercing; C_{pt} is cost of one piercing.

If various piercing types are used then (2) becomes:

$$t_{cut} = L_{on} \div V_{on} + L_{off} \div V_{off} + \sum_{j=1}^{p} N_{pt}^{j} \cdot t_{pt}^{j} \qquad (4)$$

when p is number of used piercing types; N_{pt}^{j} is numbers of j type piercing; t_{pt}^{j} is time of one j type piercing. The cutting cost F_{cost} is calculated by:

$$F_{\cos t} = L_{off} \cdot C_{off} + L_{on} \cdot C_{on} + \sum_{j=1}^{p} N_{pt}^{j} \cdot C_{pt}^{j}, \qquad (5)$$

when C_{pt}^{j} is cost of one j type piercing.

In (2)-(5) the values of speed of working and idling tool path, time of one piercing are usually constant for concrete material and manufacturing equipment. If standard cutting technique (when numbers of piercing equal numbers of cutting contours) is applied then numbers of piercing are constant too.In (3) and in (5) C_{on}, C_{off} and C_{pt} are values depending on type of CNC cutting machines, cutting technologies, thickness and type of material.

Any objective function (2)-(5) for the tool path depends on elements of tuple (1). Further we will consider the main technological restrictions for admissible values of elements $M_1, M_1^*, \ldots, M_K, M_K^*, i_1, \ldots, i_K$.

3. FORMALIZATION OF CONSTRAINTS FOR THE TOOL PATH

3.1. Constraints for coordinates of piercings

This type of constraints is determined by cutting technology features. Piercing shall occur at sufficient distance from part contour. The distance is defined by technological parameters. Coordinates of piercings $M_k; k = \overline{1,K}$ are calculated to fall into admissible geometrical area. Fig.9 shows an admissible geometrical area of piercings for external contours of parts A_2, A_{12} and A_{13}. A minimum admissible distance from a contour of part upto piercing is equal to 25 mm.

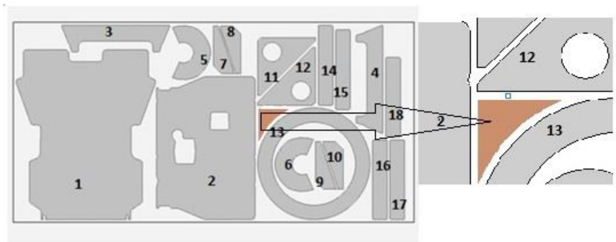

Figure 9. Example of admissible geometrical area for piercings (brown color)

By E_j^d $(j=\overline{1,N})$ we will denote the equidistant contours of contours C_j, remote from them at a distance d ($E_j^d \subset \mathbb{R}^2$). E_j^d are external equidistant contours for external contours of parts and at same time E_j^d are internal equidistant contours for internal contours of parts.

Let *OUT* is a set of indexes of external contours, and *IN* is respectively a set of indexes of internal contours $(OUT=\{j_1,j_2,...,j_l\}; IN=\{q_1,q_2,...,q_s\})$, i.e. $OUT \subseteq \{\overline{1,N}\}; IN \subset \{\overline{1,N}\}$. Let's notice that if $l=N$ (all contours are external) then $IN=\varnothing$. Let ∂ is a half with (a half allowance) for the cutting process. Then trajectories of the cutting segments for cutting of parts $A_1,A_2,...,A_n$ has to contain all contours E_j^{∂} $(j=\overline{1,N})$, i.e. $\bigcup\limits_{j=1}^{N} E_j^{\partial} \subseteq \bigcup\limits_{k=1}^{K} B^{S_k} \subset \bigcup\limits_{k=1}^{K} S_k$.

Let $d1$ is a minimum admissible distance from a equidistant contour E_j^{∂} upto any piercing. By P_j^{Δ} we will denote the two-dimensional geometrical objects restricted by closed contours E_j^{Δ} $(j=\overline{1,N})$. Then piercings and points of switching the tool off for each cutting segment S_k shall meet the following conditions:

$$\forall k \in \{\overline{1,K}\} \;\; M_k \in (B \setminus \bigcup_{j\in OUT} P_j^{\partial+d1}) \bigcup_{q\in IN} P_q^{\partial+d1} \qquad (6)$$

$$M_k^* \in \left(B \setminus \bigcup_{j\in OUT} P_j^{\partial}\right) \bigcup_{q\in IN} P_q^{\partial} \qquad (7)$$

As it is easy to notice the sets meeting conditions (6) and (7) have cardinality of continuum. At the same time the main approach to the solution of optimizing tasks (2)-(5) is a reducing of set of admissible tool paths upto discrete set. In our formulation it means need of choice of discrete subset of admissible values of elements for tuple (1). The easiest way for the solution of this task at the choice of admissible values for M_K, M_K^* is formation of the discrete set on borders of geometrical areas (6) and (7). In Fig. 10 finite set of admissible piercings for the reviewed example is shown.

Value of *d1* (minimum admissible distance from a basic segment upto any piercing) is equal to 25 mm.

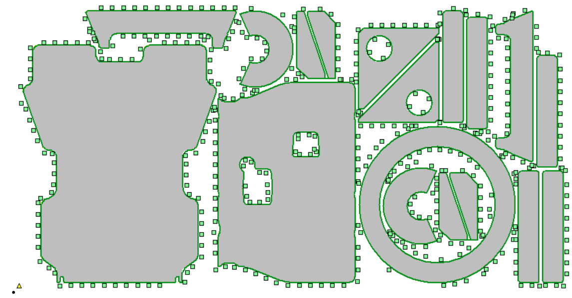

Figure 10. Example of finite set of piercings (green color)

3.2. A constrain "A part hardness rule".

This kind of the additional restrictions for values of the piercing coordinates is formulated by Petunin (2009) in form of the heuristic recommendation for the technologists-programmers developing the NC programs. It is caused by thermal deformations of material in case of thermal cutting of parts. Its sense consists in the following.

The place of piercing and the direction of the cutting are chosen so that at first the contour sites located in close proximity to material border, or to border of the cut-out area were cut, and completion of the cutting happened on a site of the contour adjoining on the "hard" part of the sheet.

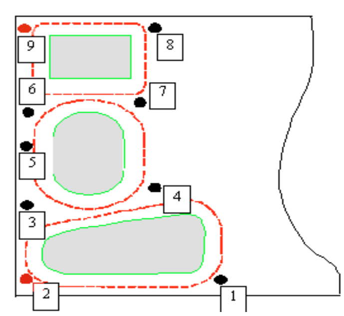

Figure 11. Illustration of "A part hardness rule" at choice of admissible piercings for 3 contours

Fig. 11 shows an Example with 3 parts (3 contours) and with 9 possible piercings.

If all contours are cutting clockwise, then the set of piercings 1, 4, 7 is the most preferable. If contour is cutting counterclockwise then piercings 4, 7, 8 (or 4,6,8) are admissible. Points 2,9 are inadmissible under any conditions.

The following formalization of the rule is offered. For each point M_k^*, meeting a condition (7) and taking into account the direction of the cutting we form two-dimensional area (hardness area) bounded by a basic segment with length of L and by an equidistant contour E^R where R – the area radius. From two other parties the area is bounded by pieces of straight lines, perpendicular to a basic segment. One of these pieces begins in a point of switching off of the tool (see Fig. 12). On this example for the cutting of a part *A4* two possible piercings *M1* and *M2* and the points of switching off of the tool corresponding to them are allocated. The formal rule of admissibility of the chosen couple (piercing, a point of switching off of the tool) is that the created hardness area (yellow color) should not have nonempty crossings with already cut out parts or with boundaries of the sheet material. In the given example the cutting segment from the beginning in *M1* point meets this requirement. For a segment from the beginning in the *M2* the respective hardness area has nonempty crossing with part *A1* but the part was already deleted from a sheet.

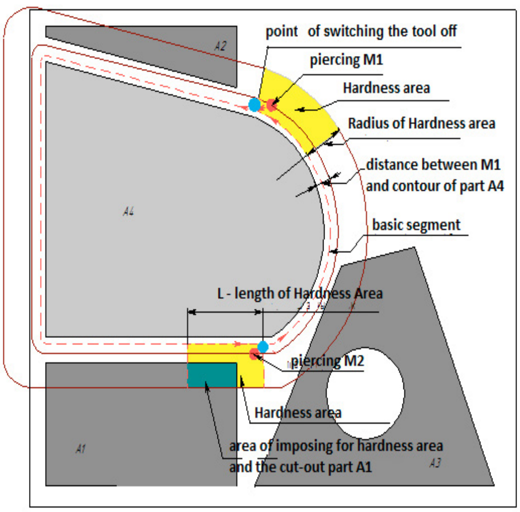

Figure 12. Geometric formalization of "A part hardness rule"

3.3. Precedence constrains

This kind of constraints imposes the restrictions on the sequence of the cutting segments $i_1,\ldots,i_K$. Constraint is caused by technological features of CNC cutting machines that do not allow cutting the internal contour accurately if external contour has already been cut because the part after the cutting can change its position on the cutting table.

A constraint is described in many publications concerning routing algorithm development (see, for Example, Verhoturov and Tarasenko (2008), Dewil et al. (2011)). Formalization and realization of this constraint usually does not causes technical difficulties. Petunin and al. (2014) shows that reduction of admissible permutations $i_1,\ldots,i_K$ allows in some problems of big dimension to use algorithms of global optimization for searching exact solutions.

3.4. A constrain "A sheet hardness rule".

This constraint imposes the restrictions also on the sequence of the cutting segments $i_1,\ldots,i_K$. Constraint is caused by technological features of thermal cutting (Petunin (2009)). The rule is a set of heuristic recommendations for the choice of an order of the cutting and is used mainly at interactive mode of the NC programs generation. In Fig.13 some simple recommendations about the choice of the sheet party with which it is necessary to begin the cutting process are shown.

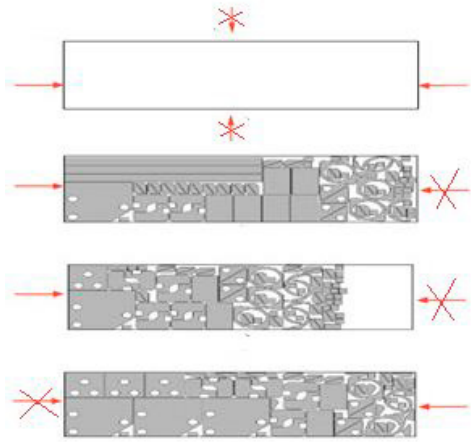

Figure 13. Rules of the choice of the sheet party at thermal cutting

A sheet hardness rule also contains many other recommendations of the concerning methods of reduction of thermal deformations. Mathematical formalization of the rule is the complex problem. The mechanism of the accounting of similar constraint was offered by Chentsov A.A. and Chentsov A.G. (2013) for the solution of the megalopolises problem (see also Chentsov and Salii (2015)). When developing algorithms of optimization for tasks (2)-(5) we used this model that allowed to formalize the constraint 3.4.

4. DISCRETE MODELS OF OPTIMIZATION PROBLEMS FOR THE TOOL PATH ROUTING

As the main optimization problem we consider *Segment Continuous Cutting Problem (SCCP)* for objective functions (2)-(5) (see Petunin (2014)). SCCP assumes that for the received nesting the number K of the used segments is in advance defined and basic segments are defined. Thus the optimizing problem of searching of tuple (1) optimizing objective function is reduced to searching of points M_k, M_k^* and also to searching of optimum sequence $i_1,\ldots,i_K$. It is easy to see, that this problem belongs to the class of the discrete optimization and at the same time to a class of the continuous optimization. Transition to the discrete model is carried out due to selection of finite set of admissible piercings M_k and points of switching off of the tool M_k^* for each given basic segment B^{S_k} ; $k=\overline{1,K}$. Selection procedure is described in paragraph 3.1. We assume that for each point M_k of possible piercings one point of switching off of the tool M_k^* is defined. Thus the problem of SCCP can be interpreted as the generalized traveling salesman problem (GTSP) with additional constraints 3.2. - 3.4. For the solution of GTSP the mathematical model of megalopolises based on the special scheme of a dynamic programming offered by A.Chentsov is used. The following algorithms for solution of SCCP are developed:

- Exact algorithm based on a dynamic programming;
- Genetic algorithm;
- Iterative greedy algorithm;

- Special option of iterative greedy algorithm for the accounting of a constraint 3.4.

All algorithms allow to consider the constraints 3.1 and 3.2. The iterative algorithm includes the accounting of constraint 3.3. Special option of algorithm considers all constraints. Now we develop the precise algorithm considering all technological restrictions of the thermal cutting. In fig. 14 the example of the exact solution of discrete option of SCCP in form of GTSP with additional constraints for objective function (2) is given. Fig. 15 shows the tool path for case of thermal technology of the cutting.

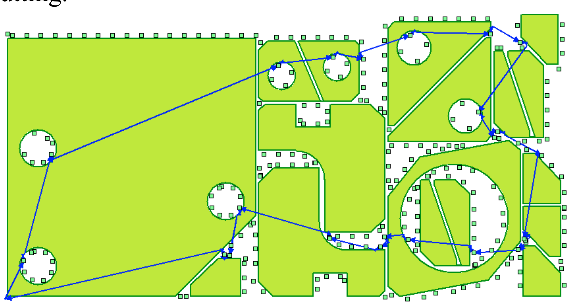

Fig.14. Example of the optimum tool path for SCCP

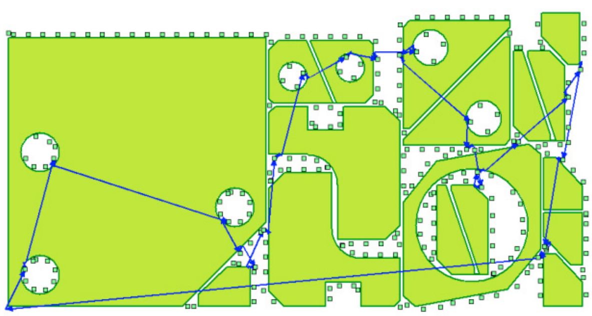

Fig.15. Example of the tool path for the thermal cutting

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